Published by Institute of Physics Publishing for SISSA/ISAS
Received: *November 11, 2004*
Accepted: *December 3, 2004*

Exactly soluble dynamics of (p, q) string near macroscopic fundamental strings

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Abstract: We study dynamics of type-IIB bound-state of a Dirichlet string and n fundamental strings in the background of N fundamental strings. Because of supergravity potential, the bound-state string is pulled to the background fundamental strings, whose motion is described by open string rolling radion field. The string coupling can be made controllably weak and, in the limit $1 \ll g_{\rm st}^2 n \ll g_{\rm st}^2 N$, the bound-state energy involved is small compared to the string scale. We thus propose rolling dynamics of open string radion in this system as an exactly solvable analog for rolling dynamics of open string tachyon in decaying D-brane. The dynamics bears a novel feature that the worldsheet electric field increases monotonically to the critical value as the bound-state string falls into the background string. Close to the background string, D string constituent inside the bound-state string decouples from fundamental string constituents.

Keywords: p-branes, D-branes, Tachyon Condensation.

Contents

1. Introduction

Decay of unstable D-branes [1], whose consequences led to deep insight into the string theory [2], is triggered by rolling the open string tachyon [3]. While there has been considerable development in recent years, the very fact that intrinsically stringy dynamics is involved has deterred much progress. For one thing, the tachyon mass is intrinsically of string scale, rendering analysis based on low-energy Born-Infeld or Yang-Mills effective field theory descriptions incomplete. Much insight to the dynamics was gained [3, 4] after the full-fledged string theoretic description — such as the boundary-state for rolling tachyon — was available. Even in this approach, variety of issues are left unanswered, especially, concerning final state of decaying D-branes [3, 5], and it would be highly desirable to devise a setup in which essential physics of rolling tachyon are retained yet rolling dynamics can be studied in a relatively simpler manner.

A situation in which dynamics involved is quite analogous to the tachyon rolling is the formation of non-threshold bound-states [6, 7]. Of particular interest are the bound-states formed out of NS-branes (either 5-branes or fundamental strings) and D-branes [6, 8].

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During the process, unlike other non-threshold bound-states formed purely out of D-branes, no open string tachyon field is involved. Rather, the constituents are simply pulled into each other. We will refer to such dynamics as *rolling radion*, where the radion refers to (one of) the transverse scalar fields measuring the relative separation between binary constituents of the bound-state.

In this work, we shall demonstrate that there exists a dynamically clean system in which the radion potential, which is the direct analog of the tachyon potential, is controllably small so that the energy would be released entirely into *massless* open or closed string excitations. The system involves fundamental (F) strings and D strings,[1] and is given by a probe $(n, 1)$ string (involving n F-strings and one D-string) moving in the background of N F-strings. Dynamics of the rolling radion is describable by $(1 + 1)$-dimensional U(1) Born-Infeld theory in the superselection sector of n unit of displacement flux.

There are several highly nontrivial and remarkable features to the system. First of all, supergravity solution describing N F-strings shows that the string coupling decreases monotonically toward the F-strings so it can be made small everywhere so that string perturbation theory is reliable. Second, NS-NS B-field increases monotonically to the critical value toward the F-strings so the test $(n, 1)$ string worldsheet feels *time-dependent* electric field as the radion rolls down. These are distinguished features never seen in tachyon rolling of unstable D-branes or in other radion rolling involving NS5-branes or D-branes. Third, in the limit

$$g_{\rm st}^2 N \gg g_{\rm st}^2 n \gg 1\,, \tag{1.1}$$

height of the radion potential, set by the binding energy density, is estimated to be order[2] of

$$\Delta\mathcal{V} = \frac{1}{g_{\rm st}^2(n+N)}\,. \tag{1.2}$$

This can be made arbitrarily small compared to the string scale by taking both the compactified dimension fixed in string unit and $g_{\rm st}^2 n$ sufficiently large. As such, all the energy released would be carried off by the radiation of the type-IIB supergravity fields.

We have organized this paper as follows. In section 2, we set out the Born-Infeld action of the $(n, 1)$ string in the background of N F-strings. In section 3, we study detailed dynamics of the radion rolling. In section 4, extend the analysis by including worldsheet modulation of the bound-state string. In section 5, we dwell on an exceptional situation, where the macroscopic F-strings are probed by a supertube — a polarized bound-state of D-particles and F-strings.

[1] It is straightforward to extend this system to various other S- or T-dual configurations. The resulting dynamics of rolling radion would exhibit essentially the same behavior.

[2] Throughout the paper, we shall adopt the convention setting 2π times the string scale squared $2\pi\ell_{\rm st}^2 \equiv 2\pi\alpha'$ to unity, and measure all dimensionful quantities in this unit. We will also suppress length of the compactified direction L_1 by setting it to unity. These dimensionful parameters can be recovered by rescaling worldsheet variables. We shall denote the Dp-brane tension as τ_p.

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2. Setup

2.1 Supergravity background of macroscopic F-strings

Consider type-IIB string theory compactified on a circle $\mathbb{S}_1$ of circumference L_1 of macroscopic size, $L_1 \gg 1$, and a closed string wrapped N times around the circle $\mathbb{S}_1$. Choosing the compactified coordinate to be x^1, the N-times wrapped F-string gives rise to the (super)gravity background in string frame as [9]

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{H(r)}\Big(- \mathrm{d}t^2 + \mathrm{d}x_1^2 \Big) + \mathrm{d}\mathbf{x}^2 \\
B_{01} &= \frac{1}{H(r)} - 1 \\
e^{2\phi} &= g_{\rm st}^2 \frac{1}{H(r)}
\end{aligned}
\tag{2.1}
$$

where $\mathbf{x}$ denotes spatial coordinates transverse to the macroscopic string, $r \equiv |\mathbf{x}|$, and the harmonic function $H(\mathbf{x})$ solving the transverse Laplace equation is given in the supergravity regime by

$$
H(r) = \left(1 + \frac{g_{\rm st}^2 N}{r^6}\right) \qquad \text{where} \qquad g_{\rm st}^2 N \gg 1\,. \tag{2.2}
$$

Compared to other p-branes in string theory [10], the background exhibits several distinguishing features. First, as one zooms into near horizon, $r \to 0$, both the string coupling $e^{\phi(r)}$ and the NS-NS B-field decrease monotonically: the string coupling interpolates from $g_{\rm st}$ to 0, and the electric component of the B-field interpolates from 0 to -1. This implies that, in the supergravity background eq. (2.1), D-brane dynamics is accessible in string perturbation theory and describable entirely in terms of noncritical open strings [11, 12, 13] near the horizon of the macroscopic F-strings.

2.2 Dirac-Born-Infeld action of (p, q) string

We are interested in the motion of (p, q) string in the background of the macroscopic F-string. For simplicity and definiteness, we shall take $(p, q) = (n, 1)$, viz. bound-state of n F-strings and a single D-string. We shall also take the F-string charge n to be in the range $1 \ll g_{\rm st}^2 n \ll g_{\rm st}^2 N$ for reasons that will become clearer momentarily. At low-energy, dynamics of the bound-state string is describable by Dirac-Born-Infeld (DBI) action in the background of eq. (2.1). Denote the bound-state string worldsheet coordinates as σ^m ($m = 0, 1$), and the worldsheet fields as X^M ($M = 0, 1, \ldots, 9$) and $\mathcal{A}_m$ ($m = 0, 1$). The DBI action then reads

$$
S_{\rm DBI} = -\tau_1 \int \mathrm{d}^2\sigma\, e^{-(\phi - \phi_\infty)} \sqrt{- \det(X^*(G + B)_{mn} + \mathcal{F}_{mn})}\,, \tag{2.3}
$$

where the pull-backs $X^*(G + B)$ are

$$
\begin{aligned}
(X^* G)_{mn} &= \frac{\partial X^M}{\partial \sigma^m} \frac{\partial X^N}{\partial \sigma^n} G_{MN}(X)\,, \\
(X^* B)_{mn} &= \frac{\partial X^M}{\partial \sigma^m} \frac{\partial X^N}{\partial \sigma^n} B_{MN}(X)\,.
\end{aligned}
\tag{2.4}
$$

The DBI action is invariant under the reparametrization of the worldsheet coordinates $\sigma^m \to \widetilde{\sigma}^m(\sigma)$ and under the U(1) gauge transformation $\mathcal{A}_m \to \mathcal{A}'_m = \mathcal{A}_m + \partial_m \epsilon$. We shall fix the reparametrization invariance by choosing the static gauge $\sigma^0 = t = X^0$, $\sigma^1 = X^1$, and fix the gauge invariance by choosing the Coulomb gauge $\mathcal{A}_0 = 0$.

For the moment, we shall study homogeneous configurations in which worldsheet fields are independent of σ^1. Because of the Coulomb gauge chosen, $\mathcal{F}_{01} = \dot{\mathcal{A}}_1 \equiv \mathcal{E}$ and the Gauss' law constraint

$$\frac{\partial}{\partial \sigma^1}\left(\frac{\delta \mathcal{L}_{\rm DBI}}{\delta \mathcal{E}}\right) = 0$$

is trivially satisfied because of homogeneity along σ^1. The DBI action is then reduced to

$$S_{\rm DBI} \equiv \int {\rm d}t{\rm d}\sigma^1 \mathcal{L}_{\rm DBI} = -\tau_1 \int {\rm d}t{\rm d}\sigma^1 \sqrt{\frac{1}{H} - \dot{\mathbf{X}}^2 - \frac{1}{H}\left((H-1) - H\mathcal{E}\right)^2}\,. \tag{2.5}$$

The momentum density $\mathbf{\Pi}_i$ conjugate to $\mathbf{X}^i$ is given by

$$\mathbf{\Pi} := \frac{\delta S_{\rm DBI}}{\delta \dot{\mathbf{X}}} = \tau_1 \frac{\sqrt{H}\,\dot{\mathbf{X}}}{\sqrt{1 - H\dot{\mathbf{X}}^2 - \left((H-1) - H\mathcal{E}\right)^2}}\,, \tag{2.6}$$

The momentum density conjugate to $\mathcal{A}_1$ is given by the displacement field:

$$\mathcal{D} := \frac{\delta S_{\rm DBI}}{\delta \dot{A}_1} = -\tau_1 \frac{\sqrt{H}\left((H-1) - H\mathcal{E}\right)}{\sqrt{1 - H\dot{\mathbf{X}}^2 - \left((H-1) - H\mathcal{E}\right)^2}}\,. \tag{2.7}$$

The displacement field $\mathcal{D}$ is conserved and is directly related to the F-string charge density inside the D-string. The latter relation follows from the observation that in general F-string current tensor $J^{MN}(x)$ couples minimally to the NS-NS B-field:

$$\Delta S_{\rm spacetime} = \int_{\mathcal{M}_{10}} {\rm d}^{10}x\, B_{MN}(x)\Big(J^{MN}(x) + \cdots\Big) \tag{2.8}$$

where the ellipses abbreviate contributions from other spacetime fields and, in particular, D-branes if present. Combining this with the fact that the B-field couples to a D-brane through the gauge-invariant combination $(X^*B + \mathcal{F})_{mn}$ and that the proportionality constant in the minimal coupling eq. (2.8) is set by the F-string tension (which we set to unity), one finds that the F-string charge density J^{01} agrees precisely with the displacement field eq. (2.7). The DBI action eq. (2.3) describes dynamics of the $(p,q) = (n,1)$ string, so the the F-string charge n should be identified with the displacement field $\mathcal{D}$:

$$\mathcal{D} = n\,.$$

2.3 Energy density and radion potential

From the canonical momenta, the hamiltonian density $\mathcal{H}$ is given by

$$\begin{aligned}\mathcal{H} &:= \mathbf{\Pi}\cdot\dot{\mathbf{X}} + \mathcal{D}\,\dot{\mathcal{A}}_1 - \mathcal{L}_{\rm DBI} \\ &= \tau_1 \frac{1}{\sqrt{H}} \frac{1 - (H-1)\left((H-1) - H\mathcal{E}\right)}{\sqrt{1 - H\dot{\mathbf{X}}^2 - \left((H-1) - H\mathcal{E}\right)^2}}\,.\end{aligned} \tag{2.9}$$

For homogeneous configuration, the canonical momenta and the time derivative of field variables are related by

$$\begin{aligned}\mathbf{\Pi}^2 &= \tau_1^2 \frac{H\,\dot{\mathbf{X}}^2}{1 - H\dot{\mathbf{X}}^2 - ((H-1) - H\mathcal{E})^2} \\ \mathcal{D}^2 &= \tau_1^2 \frac{H\left((H-1) - H\mathcal{E}\right)^2}{1 - H\dot{\mathbf{X}}^2 - ((H-1) - H\mathcal{E})^2}\,.\end{aligned} \tag{2.10}$$

These relations facilitate simplifying the hamiltonian density into the following forms:

$$\begin{aligned}\mathcal{H} &= \frac{1}{H}\left(\frac{\sqrt{H\tau_1^2 + \mathcal{D}^2}}{\sqrt{1 - H\dot{\mathbf{X}}^2}} + (H-1)\mathcal{D}\right) \\ &= \frac{1}{H}\left(\sqrt{H\left(\tau_1^2 + \mathbf{\Pi}^2\right) + \mathcal{D}^2} + (H-1)\mathcal{D}\right)\,.\end{aligned} \tag{2.11}$$

Though the result eq. (2.11) is elementary, one can extract a lot of physics intuitions. We enlist some of them here.

- Consider two extreme limits of F-string charge n. For nearly D-string case, the energy density scales as τ_1 times $1/\sqrt{H}$. This is as it should be since the DBI action is proportional to $e^{-\phi}\sqrt{G_{00}G_{11}} \sim 1/\sqrt{H}$. For nearly F-string, the energy density scales as $\mathcal{D}$. Again, this is as it should be since the first term's leading contribution is $H^{-1}\mathcal{D}$ and cancels off part of the second term's contribution $-(H^{-1}-1)\,\mathcal{D}$, thus scaling as $\mathcal{D}$.

- Static energy of the $(n,1)$-string is obtained from eq. (2.11) by setting $\mathbf{\Pi} = 0$, which we will refer as 'radion potential'. As shown in figure 1, the radion potential is a monotonic function of r, equivalently, $H(r)$.

At asymptotic infinity $r = \infty$, $H(r) = 1$, the energy density is given by $\sqrt{\tau_1^2 + \mathcal{D}^2}$. As the center is approached $r \to 0$, $H(r) \to \infty$, the energy density scales to $\mathcal{D}$. The monotonic behavior demonstrates that the $(n,1)$ string can lower its energy density simply by sliding down to the center, viz. toward the macroscopic F-strings. Once the $(n,1)$-string meets the macroscopic F-string, it will form a non-threshold bound-state of $(n+N,1)$-string. In the limit $g_{\rm st}^2 N \gg g_{\rm st}^2 n \gg 1$, the binding energy of a D-string to $(N+n)$ multiply wound F-string is extremely small, since

$$\sqrt{\tau_1^2 + (N+n)^2} = \left[(N+n) + \frac{1}{2}\frac{1}{g_{\rm st}^2}\frac{1}{(N+n)} + \cdots\right],$$

and the second term is of order $\mathcal{O}(1/g_{\rm st}^2(N+n))$ and can be made sufficiently small by taking the limit

$$g_{\rm st}^2(N+n) \gg 1 \qquad \text{and} \qquad g_{\rm st} \ll 1\,. \tag{2.12}$$

Note that this remains true for any length of the compactified dimension as long as it is finite. Once the $(n,1)$ string binds with the background N F-strings, the released binding

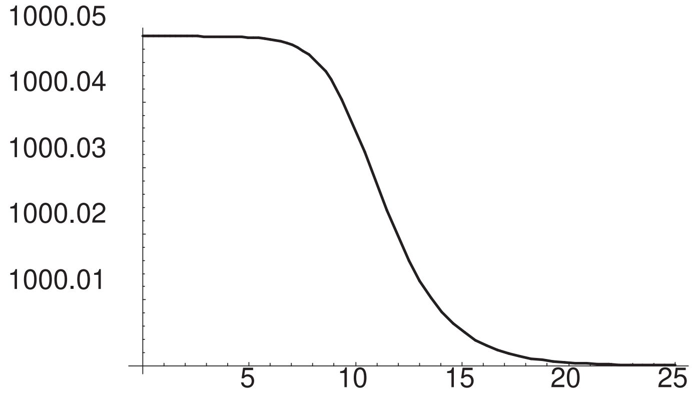

Figure 1: The static radion potential of $(n,1)$-string in the background of N macroscopic F-string for $g_{\rm st} = 0.1$ and $n = 1000$. The horizontal axis is $\log H(r)$ and the vertical axis is the static energy density measured in unit of the F-string scale. The binding energy of $(n,1)$-string (as measured by the difference of the two asymptotic plateau values) is parametrically small, and is about 5×10^{-5} of the original/final energy. Radion rolls down the potential hill and release the binding energy either into 'radion matter' or into radiation of supergravity modes.

energy ΔE would be

$$\begin{aligned}\Delta E &= \left\{\left[N + \left(n + \frac{1}{2}\frac{1}{g_{\rm st}^2}\frac{1}{n} + \cdots\right)\right] - \left[(N+n) + \frac{1}{2}\frac{1}{g_{\rm st}^2}\frac{1}{N+n} + \cdots\right]\right\} \\ &\sim \mathcal{O}\left(\frac{1}{g_{\rm st}^2 n}\right),\end{aligned} \tag{2.13}$$

thus setting the height of the radion static potential, eq. (1.2). We can render the total released energy sufficiently small compared to the string scale and the dynamics perturbative by taking the limit

$$g_{\rm st}^2 n \gg 1 \qquad \text{and} \qquad g_{\rm st} \ll 1\,. \tag{2.14}$$

Therefore, under this circumstance, emission of massive closed string states would be kinematically forbidden, and the entire process of the radion rolling of $(n,1)$-string in the macroscopic F-string background is amenable completely within the regime of low-energy supergravity approximation!

Combining the two conditions eqs. (2.12), (2.14), we are led to consider the limit alluded already in the introduction.

- One novel aspect of the radion rolling process is that the worldvolume electric field is not a constant but ought to change simply because the pullback of NS-NS B-field varies radially and because the displacement is already fixed by the F-string charge. To see how the electric field behaves, consider the displacement field near the asymptotic infinity and the center. Near the asymptotic infinity, $r \to \infty$ and $H \to 1$, the displacement field scales as

$$\mathcal{D} \longrightarrow \frac{1}{g_{\rm st}}\frac{\mathcal{E}_\infty}{\sqrt{1-\mathcal{E}_\infty^2}}\,. \tag{2.15}$$

Thus, the electric field on the D-string sitting at asymptotic infinity would take a value near the critical field $\mathcal{E}_\infty \sim \mathcal{O}(1)$. This value (hence the critical electric field minus this value) is set by the net F-string charge. Now, as the radion rolls down toward the macroscopic F-string, the electric field gets increased and attain precisely the critical value at the moment the $(n, 1)$-string hits the macroscopic F-string. This is readily seen from eqs. (2.7), (2.9): the displacement field may be expressed as

$$\frac{D}{H^2} = -\left(\mathcal{H} - \mathcal{D}(1 - H^{-1})\right)\left((1 - H^{-1}) - \mathcal{E}\right),$$

so, in case the displacement field $\mathcal{D}$ is uniform on the worldsheet, the electric field approaches the critical value 1 as

$$\mathcal{E} \to \left(1 - \mathcal{O}\left(\frac{1}{H}\right)\right). \tag{2.16}$$

It shows that the $(n, 1)$-string turns into a *tensionless* noncritical string as the radion rolls down to the macroscopic F-strings.[3]

2.4 Constraints and conserved quantities

Before proceeding further, we shall discuss conserved quantities during the homogeneous radion rolling process. By construction, the DBI action is invariant under the worldsheet reparametrization and gauge transformation. In the previous section, we have fixed them by choosing the static gauge for the reparametrization invariance and the Coulomb gauge for the gauge invariance. As such, the dynamics described in terms of the gauge-fixed DBI action ought to be supplemented by the corresponding constraints. Following the methods originally developed in [14], we now demonstrate that these constraints lead to conserved charges.

For the gauge invariance, the constraint gives rise to the Gauss' law:

$$\partial_\sigma\left(\frac{\delta S_{\rm DBI}}{\delta \mathcal{E}}\right) = \partial_\sigma \mathcal{D} = 0\,.$$

It implies that the displacement field is constant-valued along the $(1, n)$-string, and is a conserved quantity. For homogeneous worldsheet field configurations, this is automatically satisfied.

For the reparametrization invariance, the constraints are obtainable by taking the DBI action *prior* to the gauge-fixing

$$S_{\rm DBI} = -\tau_1 \int {\rm d}t{\rm d}\sigma\, \sqrt{-H(\mathcal{G}_{00}\mathcal{G}_{11} - \mathcal{G}_{01}\mathcal{G}_{10})}\,, \tag{2.17}$$

where

$$\begin{aligned}
\mathcal{G}_{00} &:= \frac{1}{H}\left(-\dot{X}_0^2 + \dot{X}_1^2\right) + \dot{\mathbf{X}}^2 \\
\mathcal{G}_{11} &:= \frac{1}{H}\left(-X_0'^2 + X_1'^2\right) + \mathbf{X}'^2
\end{aligned}$$

[3] Actually, the conclusion is more general and holds also for $n = 0$.

$$
\begin{aligned}
\mathcal{G}_{01} &:= -\left(\frac{1}{H}-1\right)\left(\dot{X}_0 X_1' - X_0'\dot{X}_1\right) + \frac{1}{H}\left(-\dot{X}_0 X_0' + \dot{X}_1 X_1'\right) + \dot{\mathbf{X}}\cdot\mathbf{X}' + \mathcal{E} \\
\mathcal{G}_{10} &:= +\left(\frac{1}{H}-1\right)\left(\dot{X}_0 X_1' - X_0'\dot{X}_1\right) + \frac{1}{H}\left(-\dot{X}_0 X_0' + \dot{X}_1 X_1'\right) + \dot{\mathbf{X}}\cdot\mathbf{X}' - \mathcal{E}
\end{aligned}
\tag{2.18}
$$

and deriving the field equations of motion for $X^0 = -X_0$ and X^1.

For X^0, the equation of motion reads

$$
\partial_t T^{00} + \partial_\sigma T^{10} = 0\,,
$$

where

$$
\begin{aligned}
T^{00} &= \frac{\tau_1\left[-\dot{X}_0\mathcal{G}_{11} + \frac{1}{2}(H-1)X_1'(\mathcal{G}_{01}-\mathcal{G}_{10}) + \frac{1}{2}X_0'(\mathcal{G}_{01}+\mathcal{G}_{10})\right]}{\sqrt{-H(\mathcal{G}_{00}\mathcal{G}_{11}-\mathcal{G}_{01}\mathcal{G}_{10})}} \\
T^{10} &= \frac{\tau_1\left[-X_0'\mathcal{G}_{00} - \frac{1}{2}(H-1)\dot{X}_1(\mathcal{G}_{01}-\mathcal{G}_{10}) + \frac{1}{2}\dot{X}_0(\mathcal{G}_{01}+\mathcal{G}_{10})\right]}{\sqrt{-H(\mathcal{G}_{00}\mathcal{G}_{11}-\mathcal{G}_{01}\mathcal{G}_{10})}}\,.
\end{aligned}
$$

In the static gauge $t = X^0$, $\sigma = X^1$, they become

$$
\begin{aligned}
\mathcal{H} &= \frac{\tau_1}{\sqrt{H}}\frac{\left[1 + H\mathbf{X}'^2 - (H-1)\left((H-1) - H\mathcal{E}\right)\right]}{\sqrt{(1-H\dot{\mathbf{X}}^2)(1+H\mathbf{X}'^2) - \left((H-1)-H\mathcal{E}\right)^2 + H^2(\dot{\mathbf{X}}\cdot\mathbf{X}')^2}} \\
T^{10} &= \frac{\tau_1}{\sqrt{H}}\frac{-H\,(\dot{\mathbf{X}}\cdot\mathbf{X}')}{\sqrt{(1-H\dot{\mathbf{X}}^2)(1+H\mathbf{X}'^2) - \left((H-1)-H\mathcal{E}\right)^2 + H^2(\dot{\mathbf{X}}\cdot\mathbf{X}')^2}}\,.
\end{aligned}
$$

They are nothing but the canonical energy density and the momentum density.

Repeating the analysis for X^1, the equation of motion is

$$
\partial_t T^{01} + \partial_\sigma T^{11} = 0\,,
$$

where

$$
\begin{aligned}
T^{01} &= \frac{\tau_1\left[\dot{X}_1\mathcal{G}_{11} - \frac{1}{2}(H-1)X_0'(\mathcal{G}_{01}-\mathcal{G}_{10}) - \frac{1}{2}X_1'(\mathcal{G}_{01}+\mathcal{G}_{10})\right]}{\sqrt{-H(\mathcal{G}_{00}\mathcal{G}_{11}-\mathcal{G}_{01}\mathcal{G}_{10})}} \\
T^{11} &= \frac{\tau_1\left[X_1'\mathcal{G}_{00} + \frac{1}{2}(H-1)\dot{X}_0(\mathcal{G}_{01}-\mathcal{G}_{10}) - \frac{1}{2}\dot{X}_1(\mathcal{G}_{01}+\mathcal{G}_{10})\right]}{\sqrt{-H(\mathcal{G}_{00}\mathcal{G}_{11}-\mathcal{G}_{01}\mathcal{G}_{10})}}\,.
\end{aligned}
$$

Again, in the static gauge, they are

$$
\begin{aligned}
T^{01} &= -\frac{\tau_1}{\sqrt{H}}\frac{H\,(\dot{\mathbf{X}}\cdot\mathbf{X}')}{\sqrt{(1-H\dot{\mathbf{X}}^2)(1+H\mathbf{X}'^2) - \left((H-1)-H\mathcal{E}\right)^2 + H^2(\dot{\mathbf{X}}\cdot\mathbf{X}')^2}} \\
\mathcal{P} &= -\frac{\tau_1}{\sqrt{H}}\frac{\left[1 - H\dot{\mathbf{X}}^2 - (H-1)\left((H-1) - H\mathcal{E}\right)\right]}{\sqrt{(1-H\dot{\mathbf{X}}^2)(1+H\mathbf{X}'^2) - \left((H-1)-H\mathcal{E}\right)^2 + H^2(\dot{\mathbf{X}}\cdot\mathbf{X}')^2}}\,,
\end{aligned}
\tag{2.19}
$$

which are the momentum density and the pressure.

2.5 Pressure

By the canonical method, we also obtain the pressure density on the worldsheet of $(n,1)$-string as

$$\begin{aligned}\mathcal{P} &= \frac{\partial L_{\rm DBI}}{\partial \mathbf{X}'}\cdot\mathbf{X}' + \mathcal{D}(-\dot{\mathcal{A}}_1) + \mathcal{L}_{\rm DBI}\\ &= -\frac{1}{H}\left[\sqrt{\left(H\tau_1^2+\mathcal{D}^2\right)\left(1-H\dot{\mathbf{X}}^2\right)} + (H-1)\mathcal{D}\right]\\ &= -\frac{1}{H}\left[\frac{\left(H\tau_1^2+\mathcal{D}^2\right)}{\sqrt{H\left(\tau_1^2+\mathbf{\Pi}^2\right)+\mathcal{D}^2}} + (H-1)\mathcal{D}\right],\end{aligned} \tag{2.20}$$

Notice that the pressure density always remains negative-definite.

In case of unstable D-brane, a salient feature was that the endpoint of the tachyon rolling comprises of pressureless 'tachyon matter'. Given that the radion rolling process can be mapped to an analog tachyon rolling, the pressure eq. (2.20) may analogously reduce to a pressureless endpoint constituents. This turns out roughly the case, as we will analyze in detail momentarily, except that the endpoint constituents still carries the pressure exerted by the n F-string constituents albeit reduced from the initial value.

To analyze the rolling behavior of the pressure, we begin with the conserved energy density. From eq. (2.11), one readily finds that

$$\sqrt{H\left(\tau_1^2+\mathbf{\Pi}^2\right)+\mathcal{D}^2} = H\,\mathcal{H} - (H-1)\mathcal{D}\,.$$

Utilizing the relation in eq. (2.20), the pressure can be recast in terms of conserved quantities as

$$\mathcal{P} = -\Big(1-\frac{1}{H}\Big)\mathcal{D} - \frac{1}{H}\left(\frac{H\tau_1^2+\mathcal{D}^2}{H(\mathcal{H}-\mathcal{D})+\mathcal{D}}\right). \tag{2.21}$$

It shows the following characteristic crossover scale for H

$$H_\star \equiv \left(\frac{\mathcal{D}}{\tau_1}\right)^2 \simeq g_{\rm st}^2 n^2 \gg 1\,.$$

Across the scale $H(r) \sim H_\star$, the numerator of the last term turns over the dominant contribution from either D-string or F-string constituents. The same argument holds for the denominator of the last term once we make use of the limit $g_{\rm st}^2 n \gg 1$ in eq. (2.21) and approximate $\mathcal{H}-\mathcal{D}$ as $1/(2g_{\rm st}^2\mathcal{D}) + \cdots$. Built upon these observations, one can re-express the pressure as

$$\mathcal{P} = -\mathcal{D} - \left(\frac{H_\star}{H+2H_\star}\right)\frac{1}{g_{\rm st}^2\mathcal{D}} + \cdots\,.$$

As the radion rolls down, viz. the $(n,1)$-string is attracted to the macroscopic F-string, the function $H(r)$ varies monotonically from 1 to $+\infty$. Accordingly, the coefficient of the second term (inside the parenthesis) decreases monotonically from $1/2$ to 0, demonstrating transparently that the pressure of the $(n,1)$ string decreases from $-\sqrt{\tau_1^2+\mathcal{D}^2}$ at infinite separation from the macroscopic F-string to $-\mathcal{D}$ at the horizon.

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The observation that the pressure exhibits a crossover around $H(r) \sim H_\star$ and form a radion matter of reduced pressure can be seen more intuitively as follows. Consider the difference between the energy density and the pressure density and utilize the Schwarz inequality:

$$\begin{aligned}\mathcal{H} - \mathcal{P} &= \frac{1}{H}\left[2(H-1)\mathcal{D} + \sqrt{H(\tau_1^2 + \mathbf{\Pi}^2) + \mathcal{D}^2} + \frac{(H\tau_1^2 + \mathcal{D}^2)}{\sqrt{H(\tau_1^2 + \mathbf{\Pi}^2) + \mathcal{D}^2}}\right] \\ &\geq 2\left[\mathcal{D} + \frac{1}{H}\left(-\mathcal{D} + \mathcal{D}\sqrt{\frac{H}{H_\star} + 1}\right)\right]. \end{aligned} \tag{2.22}$$

In the asymptotic region $r \to \infty$, equivalently, $H(r) \to 1$, the quantity in the last line yields $2\sqrt{g_{\rm st}^{-2} + \mathcal{D}^2}$. This is nothing but twice of the $(n,1)$-string tension. Recall that $(n,1)$-string is a BPS configuration, thus the energy density and the pressure takes the same value. Therefore the Schwarz inequality is actually saturated in the asymptotic region. Rolling inward, one readily observe that the inequality remains saturated in the region where $1 \ll H(r) \ll H_\star$. The quantity in the last line now yields

$$\begin{aligned}\mathcal{H} - \mathcal{P} &\to 2\left[\left(\mathcal{D} - \frac{\mathcal{D}}{H} + \cdots\right) + \left(\frac{\mathcal{D}}{H} + \frac{\tau_1^2}{2}\frac{1}{\mathcal{D}} + \cdots\right)\right] \\ &= 2\left[\mathcal{D} + \frac{1}{2g_{\rm st}^2}\frac{1}{\mathcal{D}} + \cdots\right], \end{aligned} \tag{2.23}$$

and is readily recognized as twice of the energy of $(1,n)$-string. Taking into consideration of the constraint $|\mathcal{P}| \leq \mathcal{E}$ due to Lorentz invariance on the string worldsheet and of the conserved energy, this implies that the pressure has not yet relaxed in the region $1 \ll H(r) \ll H_\star$. Rolling further into the near-coincidence region $H(r) \gg H_\star$, the quantity in the last line approaches $2\mathcal{D}$ from above:

$$\mathcal{H} - \mathcal{P} \to 2\left[\mathcal{D} + \frac{1}{g_{\rm st}\sqrt{H}} + \cdots\right]. \tag{2.24}$$

As the energy remains conserved throughout, by comparing the two expressions eq. (2.23) and eq. (2.24), one learns that the pressure is lowered as the $(n,1)$-string approaches to the macroscopic F-strings.

3. Radion dynamics for nonzero angular momentum

3.1 Effective potential

In general, the rolling radion dynamics between the $(n,1)$-string and the macroscopic N F-string would involve nonzero impact parameter and hence angular momentum. In this section, we shall study such situations in detail. Because of SO(8) symmetry in the transverse space of $\mathbf{X}$'s, the trajectories of the two-body motion under the SO(8) invariant central potential always lie on a plane. We shall take the plane to be along $(X^8 = R\cos\Theta, X^9 = R\sin\Theta)$. Again, we focus on homogeneous worldsheet field configurations. By repeating the canonical analysis as in section 2, one readily obtains conserved

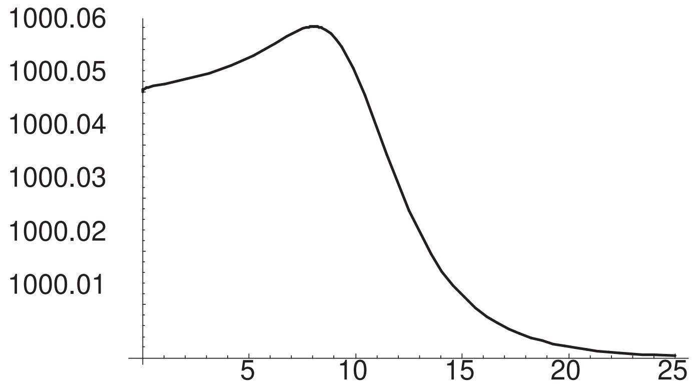

Figure 2: The effective potential of $(n, 1)$-string circulating the background of N macroscopic F-string for $g_{\rm st} = 0.1$, $n = 1000$ and $\ell^2/(g_{\rm st}^2 N^2) = 1.25$. The horizontal axis is $\log H(r)$ and the vertical axis is the static energy density measured in unit of F-string tension $\tau_{\rm F}$. Compared to the static potential at zero angular momentum, there shows up an angular momentum-dependent potential barrier. As the angular momentum ℓ is increased, the barrier gets higher.

quantities such as energy $\mathcal{H}$ and angular momentum ℓ. By SO(8) symmetry, the angular momentum ℓ

$$\ell = (R^2\dot{\Theta})\frac{\sqrt{H\tau_1^2 + \mathcal{D}^2}}{\sqrt{1 - H(\dot{R}^2 + R^2\dot{\Theta}^2)}}$$

is a conserved quantity. So, for the motion with nonzero angular momentum ℓ, the energy density $\mathcal{H}_\ell$ for motion with a fixed angular momentum ℓ is given by

$$\begin{aligned}\mathcal{H}_\ell &= \frac{1}{H}\left[\frac{\sqrt{H\tau_1^2 + \mathcal{D}^2}}{\sqrt{1 - H(\dot{R}^2 + R^2\dot{\Theta}^2)}} + (H-1)\mathcal{D}\right] \\ &= \frac{1}{H}\left[\sqrt{H\left(\tau_1^2 + {\Pi_r}^2 + \frac{\ell^2}{R^2}\right) + \mathcal{D}^2} + (H-1)\mathcal{D}\right], \end{aligned} \tag{3.1}$$

where Π_r is the conjugate momentum to R. It shows that nonzero angular momentum gives rise to the contribution of the centrifugal barrier through the first term. So, for motions with a given angular momentum ℓ, the effective potential is obtained by setting $\Pi_r = 0$ in $\mathcal{H}$. Schematic view of the effective potential is plotted in figure 2.

It shows that the angular momentum gives rise to a potential barrier between the asymptotic region and the near-center region. Since the initial radial velocity increases the energy, the effective potential implies that up to a critical radial velocity, the $(n, 1)$-string would experience centrifugal barrier. Beyond that critical velocity, the motion would simply fall into the near-center region.

3.2 Pressure and angular momentum

In the previous section, we have seen that the pressure on the $(n, 1)$-string worldsheet relaxes steadily as the string moves toward the macroscopic F-string. A pertinent question

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is to explore possible effects of nonzero angular momentum to the pressure. A priori, such effects might be of quite a complicated nature much the way the monotonic static potential in figure 1 was turned by nonzero angular momentum into an effective potential with features shown in figure 2.

In this subsection, we will show that, once the string is captured to the macroscopic F-string, the $(n, 1)$-string diminishes the pressure in a universal manner irrespective of the angular momentum. In other words, the relaxation process of the $(n, 1)$-string with nonzero angular momentum is the same as that of the $(n, 1)$-string with vanishing angular momentum. This is quite an interesting result and implies that nonzero angular momentum does not exert a barrier to the pressure, in sharp contrast to the energy.

By the canonical method, the pressure $\mathcal{P} = T_{11}$ is obtained as before. The result for nonzero angular velocity is

$$\mathcal{P} = -\frac{1}{H}\Big[\sqrt{\Big(H\tau_1^2 + \mathcal{D}^2\Big)\Big(1 - H(\dot{R}^2 + R^2\dot{\Theta}^2)\Big)} + (H-1)\mathcal{D}\Big].$$

The result is as expected — it simply expresses the radial and the azimuthal motion in two-dimensional plane on which the motion lies. Using the relations

$$\begin{aligned}\Pi_r^2 &= \tau_1^2 \frac{H\dot{R}^2}{1 - H(\dot{R}^2 + R^2\dot{\theta}^2) - \Big((H-1) - H\mathcal{E}\Big)^2}\\ \frac{\ell^2}{R^2} &= \tau_1^2 \frac{H\,R^2\dot{\theta}^2}{1 - H(\dot{R}^2 + R^2\dot{\theta}^2) - \Big((H-1) - H\mathcal{E}\Big)^2},\end{aligned}$$

one readily finds that the pressure for the motion with angular momentum ℓ is given in terms of conserved quantities as

$$\mathcal{P}_\ell = -\frac{1}{H}\left[(H-1)\mathcal{D} + \frac{H\tau_1^2 + \mathcal{D}^2}{\sqrt{H(\tau_1^2 + \Pi_r^2 + \frac{\ell^2}{R^2}) + \mathcal{D}^2}}\right]. \tag{3.2}$$

Comparing the expression with the conserved energy eq. (3.1), one finds that influence of the angular momentum on the pressure is the same as that on the energy — inside the square-root, $\mathbf{\Pi}^2$ is replaced by $(\Pi_r^2 + \ell^2/R^2)$. Therefore, as for the situation of zero angular momentum, by eliminating the square-root in favor of the conserved energy $\mathcal{H}_\ell$, we obtain

$$\mathcal{P}_\ell = -(1 - \frac{1}{H})\mathcal{D} - \frac{\Big(\tau_1^2 + \frac{1}{H}\mathcal{D}^2\Big)}{H(\mathcal{H}_\ell - \mathcal{D}) + \mathcal{D}}\,.$$

Since the above functional form is precisely the same as that for zero angular momentum once $\mathcal{H}, \mathcal{P}$ are replaced by $\mathcal{H}_\ell, \mathcal{P}_\ell$, the relaxation process of the pressure would be universal, independent of the value of the angular momentum.

3.3 Capture cross-section

Now that planar radion dynamics is governed by the effective potential exhibiting potential barrier, we shall estimate the cross-section for the $(n, 1)$-string being captured by the

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macroscopic F-string. Let the $(n,1)$-string impinge on the macroscopic F-string at $r \to \infty$ with the impact parameter L and the radial velocity V. From the expression for $\mathcal{E}$ and ℓ, it is straightforward to see, as $r \to \infty$, that the conserved quantities are

$$\mathcal{H} \to \frac{\sqrt{\tau_1^2 + \mathcal{D}^2}}{\sqrt{1-V^2}} \qquad \text{and} \qquad \ell \to \frac{\sqrt{\tau_1^2 + \mathcal{D}^2}}{\sqrt{1-V^2}}\, LV\,. \tag{3.3}$$

Notice that, for such motions, the energy is always larger than the rest mass $\sqrt{\tau_1^2 + \mathcal{D}^2}$.

How does the effective potential, as depicted in figure 2, scale with the angular momentum ℓ? As the $(n,1)$-string moves inward, the effect of nonzero angular momentum scales inversely with the radial distance-squared, and hence dominates over the effect of the harmonic function $H(r)$. Denote the location of the maximum of the effective potential as $r_\star$ and $H_\star \equiv H(r_\star)$. The effective potential behaves in ways that, as the angular momentum ℓ is cranked up, the maximum location $r_\star$ and $H_\star$ is pushed outward, and height of the potential barrier $V_{\rm eff}(H_\star) - V_{\rm eff}(0)$ is increased.

Such behavior of the effective potential entails qualitative features that, for a given angular momentum ℓ, there exists a critical energy $\mathcal{H}_{\rm cr}(\ell)$ such that the motion starting at infinity with $\mathcal{H} < \mathcal{H}_{\rm cr}$ would be bounced at some finite radial distance back to infinity. Functional form for $\mathcal{H}_{\rm cr}(\ell)$ as a function of ℓ (as well as other quantum numbers n, N) is rather complicated. Fortunately, its explicit functional form would not be needed, as it is evident that $\mathcal{H}_{\rm cr}(\ell)$ increases monotonically with ℓ simply because the effective potential itself increases monotonically by the contribution of nonzero angular momentum ℓ. Being monotonic, the relation implies that $\mathcal{H}_{\rm cr}(\ell)$ is invertible to a function $\ell_{\rm cr} \equiv \ell(\mathcal{H}_{\rm cr})$. Turning around the arguments, since we have a well-defined inverse $\ell_{\rm cr}(\mathcal{H})$, the motion would be bounced off if $\ell > \ell_{\rm cr}(\mathcal{H})$ for a given $\mathcal{H}$. Given an initial velocity V, it happens when the impact parameter L is large enough:

$$L > \frac{V^{-1}\sqrt{1-V^2}}{\sqrt{\tau_1^2 + \mathcal{D}^2}}\, \ell_{\rm cr}(\mathcal{H}_\ell) \qquad \text{where} \qquad \mathcal{H}_\ell = \frac{\sqrt{\tau_1^2 + \mathcal{D}^2}}{\sqrt{1-V^2}}\,.$$

For motions with the impact parameter less than $\ell_{\rm cr}(\mathcal{H})$, the motion of the $(n,1)$-string would reach all the way to the macroscopic F-string located at $r = 0$. In the previous section, we have shown that the pressure relaxes steadily as the radion rolls to the center $r = 0$, and eventually the $(n,1)$-string forms a bound-state with the macroscopic F-string. Therefore, we can define a total cross-section $\sigma_{\rm capture}$ for the $(n,1)$-string being captured by the macroscopic F-string as

$$\sigma_{\rm capture}(V) \;=\; \frac{16\pi^3}{105}\left(\frac{V^{-1}\sqrt{1-V^2}}{\sqrt{\tau_1^2 + \mathcal{D}^2}}\; \ell_{\rm cr}(\mathcal{H}_\ell)\right)^7,$$

where $16\pi^3/105$ is the angular volume of the transverse disc $\mathbb{S}^7$. For the scattering states, in principle it is possible to get the differential cross-section, but because analytic solution for the motions is not in our hand, it is not very illuminating to analyze further.

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4. Mapping to rolling of 'analog' tachyon

We have seen in the above analysis that a $(n, 1)$ string moving in the background of large-N F1 strings generically falls toward $r \to 0$ if angular momentum is not too large. As emphasized by Kutasov [15] and further studied in subsequent works [16], the situation bears a certain similarity with rolling tachyon dynamics of unstable D-branes. In this section, we shall push such analogy to our rolling radion dynamics in a more quantitative manner.

Begin with the radial dynamics as described by eq. (3.1),

$$\dot{R}^2 = \frac{1}{H}\left[1 - \frac{\left(\tau_1^2 + \frac{\ell^2}{R^2}\right) H + \mathcal{D}^2}{(H(\mathcal{H} - \mathcal{D}) + \mathcal{D})^2}\right]. \tag{4.1}$$

As we discussed in the previous section, for $\mathcal{H}$ and ℓ below threshold, the orbit of the bound-state string is bound to the horizon $R = 0$. Near $R \sim 0$,

$$\dot{R}^2 \sim \frac{1}{H} \sim \frac{R^6}{g_{\rm st}^2 N} \qquad \text{so} \qquad R \sim \frac{(g_{\rm st}^2 N)^{\frac{1}{4}}}{\sqrt{2}}\, t^{-\frac{1}{2}}\,.$$

It shows that, to reach the horizon at $R = 0$, the motion takes an infinite coordinate time lapse, but a finite proper time lapse.

As discussed in the previous sections, dynamics with zero angular momentum is described by one-dimensional motion, while those with nonzero angular momentum is reduced to two-dimensional motion. When mapped to analog tachyon rolling dynamics, these two situations correspond to real-valued tachyon relevant for unstable D-brane decay and to complex-valued tachyon relevant for $\mathrm{D}\overline{\mathrm{D}}$ brane-antibrane pair. We now demonstrate such correspondences.

4.1 D-string with zero angular momentum

Consider first the motion of a pure D-string (so that $\mathcal{D} = 0$) with no angular momentum $\ell = 0$. In such a situation, we can throw away the complicated term in the action involving gauge field and take a simple effective action,

$$S_{\rm D1} = -\tau_1 \int {\rm d}t\, \frac{1}{\sqrt{H}}\, \sqrt{1 - H\,\dot{R}^2}\,.$$

The motion is one-dimensional, so R can be taken any one of the 8 scalar fields $\mathbf{X}$, say, $R = X^9$. We then introduce analog 'tachyon' field T, a real-valued scalar field, as the proper variation in the target space:

$$\sqrt{H}{\rm d}R \equiv -{\rm d}T \quad \rightarrow \quad T(R) = -\int^R {\rm d}R'\, \sqrt{H(R')}\,. \tag{4.2}$$

It shows that radion dynamics for zero angular momentum is analogous to tachyon dynamics of unstable D-brane.

In the near-horizon region, $R \to 0$, the analog tachyon field moves out to infinity:

$$T(R) \sim \frac{\sqrt{g_{\rm st}^2 N}}{2} \frac{1}{R^2}\,.$$

In terms of the tachyon field eq. (4.2), the D-string action is expressible as

$$S_{\rm D1} = -\tau_1 \int {\rm d}t\, V(T)\, \sqrt{1-\dot{T}^2}\,,$$

where the 'tachyon' potential is denoted as $V(T)$. Near the horizon, $T \to \infty$, and the tachyon potential decays in power-law type:

$$V(T) \sim (g_{\rm st}^2 N)^{\frac{1}{4}} \frac{1}{(8T^3)^{\frac{1}{2}}}\,. \tag{4.3}$$

Likewise, the pressure, $\mathcal{P}$, is expressible in terms of the analog tachyon field T. Again, utilizing eq. (4.1) to eq. (2.20), one finds that the pressure asymptotes as

$$\mathcal{P} \sim -\frac{\tau_1^2}{g_{\rm st}^2 N \mathcal{H}} R^6 \sim -\frac{\tau_1^2}{8\mathcal{H}} \sqrt{g_{\rm st}^2 N} \frac{1}{T^3}\,.$$

It shows that the pressure on the D-string worldsheet vanishes in power-law. We therefore see that both the tachyon potential and the pressure decay at $T \to \infty$ much like the tachyon rolling of unstable D-branes except that the asymptotic behavior is power-like rather than behaving exponentially.

4.2 $(n,1)$-string with nonzero angular momentum

We shall now extend the mapping to more general situations with $\mathcal{D} \neq 0$ and $\ell \neq 0$. Begin with the hamiltonian eq. (2.11):

$$\mathcal{H} = \frac{1}{H}\left(\sqrt{H\left(\tau_1^2 + \mathbf{\Pi}^2\right) + \mathcal{D}^2} + (H-1)\mathcal{D}\right).$$

For the motion with nonzero angular momentum, we simply substitute in the (R,Θ)-plane as

$$\mathbf{\Pi}^2 \to \Pi_r^2 + \frac{\Pi_\theta^2}{R^2} = \Pi_r^2 + \frac{\ell^2}{R^2}\,,$$

where Π_r and Π_θ are conjugate momenta to R and Θ, respectively. We shall obtain the effective lagrangian for the motion with a fixed angular momentum ℓ by performing the Legendre transform with respect to Π_r. First solve Π_r in terms of $\dot{R}$ from

$$\dot{R} = \frac{\partial \mathcal{H}}{\partial \Pi_r} = \frac{\Pi_r}{\sqrt{H\left(\tau_1^2 + \Pi_r^2 + \frac{\ell^2}{R^2}\right) + \mathcal{D}^2}}\,.$$

We then define effective lagrangian $L_{\rm eff}$ by Legendre transform:

$$\begin{aligned}\mathcal{L}_{\rm eff} &= \Pi_r \dot{R} - \mathcal{H} \\ &= -\frac{1}{H}\left(\frac{H\,(\tau_1^2 + \frac{\ell^2}{R^2}) + \mathcal{D}^2}{\sqrt{H(\tau_1^2 + \Pi_r^2 + \frac{\ell^2}{R^2}) + \mathcal{D}^2}} + (H-1)\mathcal{D}\right) \\ &= -\frac{1}{H}\left(\sqrt{H\left(\tau_1^2 + \frac{\ell^2}{R^2}\right) + \mathcal{D}^2}\sqrt{(1 - H\,\dot{R}^2)} + (H-1)\mathcal{D}\right).\end{aligned}$$

As discussed in previous sections, dynamics for nonzero angular momentum as described by the above effective lagrangian is describable on a two-dimensional plane in **X**-space. Take the plane along $(8,9)$-directions and denote $Z \equiv (X^8 + iX^9)$. We then see that the proper variation of radial field $R \equiv |Z|$ on the plane can be mapped to a complex-valued tachyon field T by:

$$\sqrt{H}\,\mathrm{d}Z = -\mathrm{d}T\,, \qquad \sqrt{H}\,\mathrm{d}\overline{Z} = -\mathrm{d}\overline{T}\,.$$

Utilizing the planar symmetry, the effective lagrangian then takes the form

$$S_{\rm eff} = -\int \mathrm{d}t\left[\tau_1 V_{\rm eff}(T\overline{T})\sqrt{1 - |\dot{T}|^2} + W_{\rm eff}(T\overline{T})\,\mathcal{D}\right],$$

where the effective tachyon potential $V_{\rm eff}(|T|)$ and the background potential $W_{\rm eff}(|T|)$ asymptote at $|T| \to \infty$ to:

$$V_{\rm eff}(|T|) \sim \frac{\ell}{\tau_1\sqrt{g_{\rm st}^2 N}}R^2 \sim \frac{\ell}{2\tau_1}\frac{1}{|T|} \qquad \text{and} \qquad W_{\rm eff}(|T|) \sim 1\,.$$

Compared to the dynamics with zero angular momentum eq. (4.3), we learn that nonzero angular momentum makes the tachyon potential vanishes slower.

The pressure can be worked out similarly. We found that

$$\mathcal{P} \sim -\mathcal{D} + \left(\mathcal{D} - \frac{\tau_1^2}{\mathcal{H} - \mathcal{D}}\right)\frac{\sqrt{g_{\rm st}^2 N}}{8}\frac{1}{|T|^3}\,.$$

It shows that the pressure decays monotonically to that of n F-strings and that the way it decays asymptotically is *independent* of the angular momentum ℓ.

As demonstrated in the previous section, because of the angular momentum barrier, the $(n,1)$ string bounces off if it starts at sufficiently large distance with $\mathcal{H} < \mathcal{H}_{\rm cr}$. Quantum mechanically, however, for small enough $\Delta\mathcal{H} = \mathcal{H}_{\rm cr} - \mathcal{H}$, the string could tunnel to the macroscopic F-string. Reinstating the size L_1 of the compactified $\mathbb{S}_1$ direction, probability is estimated as

$$P \sim e^{-f(\Delta\mathcal{H})L_1}\,,$$

for a function f determinable from the effective potentials. This will be exponentially suppressed for large L_1.

5. Inhomogeneous radion rolling

So far, we dealt with homogeneous motion of the $(n,1)$-string. In this section, we examine how modulation of the $(n,1)$-string affects the dynamics. By modulation, we mean that shape of the $(n,1)$ string in the transverse directions is initially inhomogeneous. The effect turns out quite interesting. As the $(n,1)$-string approaches the background F-strings, the n F-string constituents become static and the D-string constituent moves freely in the background of $(N+n)$ F-strings. The modulation effect is carried solely by the D-string constituent [17].

In case the worldsheet fields $\mathbf{X}$ being functions of both t and σ, the DBI action of the $(n,1)$-string was given in eq. (2.17). Explicitly,

$$S_{\rm DBI} = -\tau_1 \int {\rm d}t{\rm d}\sigma \sqrt{H}\sqrt{\left(H^{-1} - \dot{\mathbf{X}}^2\right)\left(H^{-1} + \mathbf{X}'^2\right) + \left(\dot{\mathbf{X}}\cdot\mathbf{X}'\right)^2 - ((1-H^{-1}) - \mathcal{E})^2}\,,$$

subject to the Gauss' law constraint $\partial_\sigma(\delta S_{\rm DBI}/\delta\mathcal{E}) = 0$.

Denote the lagrangian density as $-\tau_1\mathcal{L}_{\rm DBI}$. Then, the canonical momenta are given by

$$\begin{aligned} \mathbf{\Pi} &= +\tau_1 \frac{1}{\mathcal{L}_{\rm DBI}}\left[\dot{\mathbf{X}} + H\Big(\dot{\mathbf{X}}(\mathbf{X}'\cdot\mathbf{X}') - \mathbf{X}'(\dot{\mathbf{X}}\cdot\mathbf{X}')\Big)\right] \\ \mathcal{D} &= -\tau_1 \frac{1}{\mathcal{L}_{\rm DBI}}\Big[(H-1) - H\,\mathcal{E}\Big]. \end{aligned} \tag{5.1}$$

From the equation of motion for gauge fields and the Gauss' law constraint, it follows that the displacement field $\mathcal{D}$ is constant. On the other hand, the gauge potential $\mathcal{A}_1$ and hence the electric field $\mathcal{E} = \dot{\mathcal{A}}_1$ may vary over the worldsheet as functions of both t and σ.

From the canonical momenta eq. (5.1), the following relations are easily obtainable:

$$\begin{aligned} \dot{\mathbf{X}}\cdot\mathbf{X}' &= \mathcal{L}_{\rm DBI}\frac{\mathbf{\Pi}\cdot\mathbf{X}'}{\tau_1} \\ \dot{\mathbf{X}}^2 &= \left(\frac{\mathcal{L}_{\rm DBI}}{\tau_1}\right)^2 \left(\frac{\mathbf{\Pi} + H(\mathbf{\Pi}\cdot\mathbf{X}')\mathbf{X}'}{1 + H\mathbf{X}'^2}\right)^2. \end{aligned}$$

These relations enables to express the DBI lagrangian density $\mathcal{L}_{\rm DBI}$ in a compact form

$$\mathcal{L}_{\rm DBI} = \frac{\tau_1\left(1 + H\mathbf{X}'^2\right)}{\sqrt{H}\sqrt{\left(\tau_1^2 + \frac{1}{H}\mathcal{D}^2\right)\left(1 + H\mathbf{X}'^2\right) + \mathbf{\Pi}^{\rm t}\mathcal{M}\mathbf{\Pi}}}\,, \tag{5.2}$$

where $\mathcal{M}$ denotes a (8×8) matrix of dyad form

$$\mathcal{M}_{ab} := \delta_{ab} + H\mathbf{X}'_a\mathbf{X}'_b. \tag{5.3}$$

The hamiltonian density $\mathcal{H}$ of modulated $(n,1)$-string:

$$\mathcal{H} = \mathbf{\Pi}\cdot\dot{\mathbf{X}} + D\dot{A}_1 - \mathcal{L}_{\rm DBI} \tag{5.4}$$

is then easily derivable. After straightforward computation and using the expression eq. (5.2) for $\mathcal{L}_{\rm DBI}$, we obtain

$$\mathcal{H} = \frac{1}{H}\left(+(H-1)\mathcal{D} + \sqrt{H\left(\left(\tau_1^2 + \frac{\mathcal{D}^2}{H}\right)(1 + H\mathbf{X}'^2) + \mathbf{\Pi}^{\rm t}\mathcal{M}\mathbf{\Pi}\right)}\right). \tag{5.5}$$

Compared to homogeneous rolling of the radion analyzed in the previous sections, a difference is that the hamiltonian now depend on the momenta Π through a quadratic form defined by the matrix eq. (5.3). In fact, the matrix eq. (5.3) admits a simple physical interpretation. Being modulated, the $(n, 1)$ string is no longer straight along the x^1-direction but sweeps a contour bent over to $\mathbf{X}$-directions. Thus, an infinitesimal element of the string contour sweeps $(\Delta X^1, \Delta\mathbf{X})$, whose proper length is given by

$$\sqrt{\left(\frac{1}{\sqrt{H}}\Delta X^1\right)^2 + \Delta\mathbf{X}^2}\,.$$

The consideration results in *increase* of the hamiltonian density and hence the string energy per unit ΔX_1, and *decrease* of the pressure per unit ΔX_1. Intuitively, this is explainable by the following considerations. First, the modulation has the effect of compressing in a given ΔX_1 interval more string bits, and this explains increase of the string mass density. Second, adjacent string bits are continuously disoriented such that tension of each bit, which is a vectorial quantity, adds up but with slight cancellations, and this explains decrease of the pressure. A phenomenon similar to this was also considered in the contexts of (p, q) string junction [17] and of tachyon rolling [18].

As $(n, 1)$ string rolls down toward the horizon, its worldsheet dynamics is governed by

$$\mathcal{H}_{(n,1)} \to \mathcal{D} + \mathcal{H}_{\rm D1}\,,$$

where

$$\mathcal{H}_{\rm D1} = \sqrt{\tau_1^2\left(\frac{1}{H} + \mathbf{X}'^2\right) + \frac{1}{H}\mathbf{\Pi}^{\rm t}\mathcal{M}\mathbf{\Pi}}\,.$$

The interpretation of the first term is clear; it is the tension of the initially dissolved F1 strings. It adds to the tension N of the background F-strings to yield the tension of the $(N + n, 1)$ bound state in large-N limit. Observe that the second part now is independent of any effects of gauge flux; the hamiltonian is *decoupled* into one describing n F1 strings and the other describing a D-string. We interpret this as a decoupling of F-string constituents out of D-string in $(n, 1)$ string world volume dynamics in large-N limit. In other words, we propose that the world volume theory of $(n, 1)$ string in large-N limit takes an approximately decoupled form,

$$\mathcal{H}_{(n,1)} = \mathcal{H}_{\rm F} + \sqrt{\tau_1^2\left(\frac{1}{H} + \mathbf{X}'^2\right) + \frac{1}{H}\mathbf{\Pi}^2 + \left(\mathbf{\Pi}\cdot\mathbf{X}'\right)^2} + \mathcal{O}\left(\frac{1}{n}\right), \tag{5.6}$$

where $\mathbf{X}^i$, $\mathbf{\Pi}_i$ are conjugate variables describing oscillations of the D-string inside the $(n, 1)$ bound-state, and $\mathcal{H}_{\rm F}$ encodes the energy of n F-strings.

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By Legendre transformation, we can easily get the effective action of the decoupled D1 inside $(n,1)$ string,

$$S_{\rm eff} = -\tau_1 \int {\rm d}t{\rm d}\sigma \sqrt{\frac{1}{H} - \dot{\mathbf{X}}^2 + \mathbf{X}'^2 + H\Big((\dot{\mathbf{X}}\cdot\mathbf{X}')^2 - \dot{\mathbf{X}}^2\mathbf{X}'^2\Big)} \quad .$$

Now, as the string approach the horizon, $H \to \infty$ and the last term inside the square-root forces the string modulation to evolve in a way that $\dot{\mathbf{X}}$ is proportional to $\mathbf{X}'$, and in turn that $\mathbf{\Pi}$ is oriented parallel to $\mathbf{X}'$. This implies that the asymptotic hamiltonian density of the D-string constituent is governed by

$$\mathcal{H}_{\rm D} = \sqrt{\Big(\frac{1}{H} + \mathbf{X}'^2\Big)}\sqrt{\Big(\tau_1^2 + \mathbf{\Pi}^2\Big)} \;\gg\; \frac{1}{\sqrt{H}}\sqrt{\Big(\tau_1^2 + \mathbf{\Pi}^2\Big)}\,.$$

The first factor is the aforementioned geometric jacobian of the string contour. Evidently, the hamiltonian density is enhanced by this geometric factor due to the presence of the modulation, the enhancement ratio being $\sqrt{1 + \mathbf{X}'^2/H}$.

Finally we like to mention that, at the first stage of the rolling, the modulation effect grows in general. This is due to typical tidal effect in the supergravity background as is clear clear from the static radion potential in figure 1.

6. Exceptional D-particles and supertubes

So far, we have dealt with dynamics of $(n,1)$ string in type-IIB string theory. Essentially the same consideration would be applicable for dynamics of other D-brane or bound-state of D-branes and NS-branes in type-IIA string theory. There is, however, an exceptional situation — unlike all other D-branes, D-particle does not give rise to radion rolling dynamics. This is because the complex of D-particles and F-strings is supersymmetric for arbitrary transverse separations, and the radion dynamics yields a motion along flat direction. Moreover, one can manufacture a supertube extended parallel to the macroscopic F-string. Again, such supertube is supersymmetric in the macroscopic F-string background and hence do not develop radion potential. In all these cases, the brane complex preserves one-quarter of the total 32 supersymmetries. In this section, we shall study features of these exceptional cases. Instead of directly verifying the vanishing radion potential, we shall demonstrate that a supertube [19, 20, 21] near the macroscopic F-strings does not exhibit tachyonic instabilities. D-particle is a special situation in which supertube loses F-string charge.

Supertube is a bound state of D-particles, F-strings and D2-branes in type-IIA string theory. Its worldvolume is homogeneously extended in its axial direction along which the bound F-strings are stretched. Interestingly the cross sectional shape of the supertubes may form an arbitrary curve in the transverse eight dimensions [22]. Here, we would like to show that there is no tachyonic instabilities for the supertube whose axial direction is aligned with the stretching direction X_1 of the background fundamental strings.

The low-energy dynamics of a supertube is conveniently described by the DBI action of the D2 brane with electric and magnetic gauge field excitations,

$$S_{\rm DBI} = -\tau_2 \int {\rm d}^3\sigma\, e^{-(\phi-\phi_\infty)}\sqrt{-\det(X^*(G+B)+\mathcal{F})}\,. \tag{6.1}$$

We shall proceed with partial gauge-fixing of the worldvolume reparametrization invariance by setting $\sigma_0 = t = X^0$ and $\sigma_1 = X^1$ while leaving the $\sigma_2 = \theta$ worldvolume direction may be fixed after solving equation of motion. Consider homogeneous dynamics along the longitudinal σ^1-direction. Turn on the worldvolume gauge field as

$$\mathcal{F} = \left[\mathcal{E}(\theta,t) - \left(1 - \frac{1}{H}\right)\right] \mathrm{d}t \wedge \mathrm{d}\sigma_1 + \mathcal{B}(\theta,t)\,\mathrm{d}\sigma_1 \wedge \mathrm{d}\theta$$

where specific parametrization of the electric field component is motivated by the effect of the supergravity background and simplification in describing supertube worldvolume excitations. Denoting the derivative ∂_θ as $'$, the action density is then reduced to $-\tau_2 \mathcal{L}_{\rm DBI}$, where

$$\mathcal{L}_{\rm DBI} = \sqrt{\left(\frac{1}{H} - \dot{\mathbf{X}}^2\right)\left(\mathbf{X}'^2 + H\mathcal{B}^2\right) + (\dot{\mathbf{X}}\cdot\mathbf{X}')^2 - H\mathcal{E}^2\mathbf{X}'^2 - 2H\mathcal{E}\mathcal{B}\,\dot{\mathbf{X}}\cdot\mathbf{X}'}\,. \qquad (6.2)$$

Again, we work with the hamiltonian formulation. Canonical conjugate momenta are

$$\begin{aligned}\mathbf{\Pi} &= \frac{\tau_2}{\mathcal{L}_{\rm DBI}}\left[\dot{\mathbf{X}}\left(\mathbf{X}'^2 + H\,\mathcal{B}^2\right) - \mathbf{X}'\left(\dot{\mathbf{X}}\cdot\mathbf{X}' - H\,\mathcal{E}\mathcal{B}\right)\right]\\ \mathcal{D} &= \frac{\tau_2}{\mathcal{L}_{\rm DBI}} H\left(\mathcal{E}\,\mathbf{X}' + \mathcal{B}\,\dot{\mathbf{X}}\right)\cdot\mathbf{X}'\,.\end{aligned} \qquad (6.3)$$

The hamiltonian density is then given by

$$\mathcal{H} = \mathcal{D}\left(1 - \frac{1}{H}\right) + \sqrt{\tau_2^2\left(\frac{1}{H}\mathbf{X}'^2 + \mathcal{B}^2\right) + \frac{1}{H}\mathbf{\Pi}^2 + \frac{1}{H^2}\mathcal{D}^2}\,.$$

We now rearrange terms inside the square-root into the following form [23]:

$$\mathcal{H} = \mathcal{D}\left(1 - \frac{1}{H}\right) + \sqrt{\left(\frac{\mathcal{D}}{H} + \tau_2\mathcal{B}\right)^2 + \frac{1}{H}\left(\tau_2\,|\mathbf{X}'| - \frac{\mathcal{B}\mathcal{D}}{|\mathbf{X}'|}\right)^2 + \frac{1}{H}\left(\mathbf{\Pi}^2 - \frac{\mathcal{B}^2\mathcal{D}^2}{\mathbf{X}'^2}\right)}\,. \qquad (6.4)$$

The second term inside the square root is manifestly positive definite. The third term inside the square root turns out positive definite because of the Schwarz inequality:

$$\mathbf{\Pi}^2 - \frac{\mathcal{B}^2\mathcal{D}^2}{\mathbf{X}'^2} = \frac{\tau_2^2}{\mathcal{L}_{\rm DBI}^2}\frac{\left(\mathbf{X}'^2 + H\mathcal{B}^2\right)^2}{\mathbf{X}'^2}\left(\dot{\mathbf{X}}^2\mathbf{X}'^2 - (\dot{\mathbf{X}}\cdot\mathbf{X}')^2\right) \geq 0\,. \qquad (6.5)$$

The equality holds when $\dot{\mathbf{X}}$ is proportional to $\mathbf{X}'$, which is equivalent to $\dot{\mathbf{X}} = 0$ by a suitable worldvolume coordinate transformation of θ. We see then that the hamiltonian density is bounded from below and saturated when both the second and the third terms in the square root of eq. (6.4) vanish:

$$\tau_2\mathbf{X}'^2 = \mathcal{B}\mathcal{D} \qquad \text{and} \qquad \dot{\mathbf{X}} = 0\,. \qquad (6.6)$$

Therefore, we conclude that

$$\mathcal{H} \geq \mathcal{D} + \tau_2\mathcal{B}\,. \qquad (6.7)$$

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We see that the minimum of the hamiltonian density, as given by the right-hand side, admits a clear physical interpretation. It contains mass density of the fundamental strings and that of the D0-branes and precisely matches with the energy density of the supertube! Recall that $\mathcal{D}$ is the lineal density of fundamental strings stretched along X^1 direction with respect to θ and $\mathcal{B}/(2\pi)$ corresponds to the area density of D-particle inside D2-brane worldvolume. Namely, $Q_1 = \int \mathrm{d}\theta\, \mathcal{D}$ counts the number of fundamental strings and $\frac{1}{2\pi}\int \mathrm{d}\sigma^1 \mathrm{d}\theta\, \mathcal{B}$ is the number of D-particles melted in the D2-brane.

The conditions eq. (6.6) are nothing but the BPS conditions for 1/4-supersymmetric supertubes ought to obey. Of course, one may explicitly verify that any solution of the above BPS equations satisfy the full equations of motion provided the Gauss law constraint $\partial_1 D = 0$. The construction of all BPS solutions is simple. We choose arbitrary static functions of $\mathbf{X}(\theta)$ and $B(\theta)$. Then $D(\theta)$ is constrained by eq. (6.6). Hence, the cross sectional curve is arbitrary in the transverse eight dimensions and the distribution of D0 over the θ direction is arbitrary too.

This demonstrates that the supertube may be located anywhere in the eight-dimensional transversal space and there is no tachyonic instability. From the consideration of partial supersymmetries preserved, this fact is quite expected as discussed before, but the actual demonstration that the BPS bound is actually saturated is highly nontrivial as we have seen.

Acknowledgments

S.-J.R was supported in part by the MOE BK-21 Project through SNU Physics Division Team 2, by the MOST-KOSEF Leading Scientist Grant, and by the Alexander von Humboldt Foundation through the W.F. Bessel Research Award. D.B. is supported in part by Korea Research Foundation Grant KRF-2003-070-C00011. H.-U.Y. is partly supported by grant No. R01-2003-000-10391-0 from the Basic Research Program of the Korea Science & Engineering Foundation.

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